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DYNAMICS OF VIBRATION OF A CANTILEVER UNDER LATERAL IMPACT OF AN ELASTIC LOAD-PART IV (ENERGY OF THE BAR)

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ABSTRACT. In this paper, the energy lost by the hammer, during impact (which is assumed to be the energy of the bar) is deduced, following the deductions of the general theory, as given in Parts I to III of this series of papers. Experiments carried out with a cantilever of length 95 cms., dia. 1.27 cms and material mild steel is reported. The agreement between the theory and the experiment is excellent.

It is shown by Banerjee (1966-Part II) that the velocity of the load at any instant is given by

$$V_t = \frac{dy_a}{dt} = 4v_0 \Sigma A_s \cos q_s t \qquad \dots (1)$$

or

$$\frac{V_t}{v_0} = 4\, \Sigma A_s \cos q_s t. \qquad \dots (2)$$

Initial energy of the hammer $\frac{1}{2}\, mv_0^2$

Equation (2) gives the ratio of the velocity of the hammer at any instant during impact, to the initial impinging velocity of the load. The variable time t is replaced by the duration of impact, when we require the ratio of the velocity of the hammer at the termination of contact to the initial velocity. The duration of impact is obtained as usual, by equating the expression for pressure (Pt. II Banerjee 1966) to zero for the given struck point.

The ratio of the loss of energy to the initial energy of the load is given by

$$1 - \left(\frac{V_t}{v_0}\right)^2 \qquad \dots (3)$$

It is noted here that the velocity of the hammer V, as calculated from eqn. (5) (Banerjee, 1966) at the termination of first contact is different than the rebound velocity of load in cases of multiple contacts. But in cases, where the influences of multiple contacts are meagre, the energy lost by the hammer, as calculated from eqn. (1) (Banerjee, 1966) will give sensibly accurate results.

The apparatus used is similar to that used by Banerjee in the study of lateral impact on cantilever excepting the photographic recording arrangements. The rod is struck transversely by the hammer from different distances away to achieve different impinging velocities. The arc along which the hammer swings is measured from the shadow of the outline of the hammer cast on a graduated translucent scale placed very near to it. The length of the hammer (pendulum bob) and its maximum arc of swing in each case help to measure the velocities respectively.

TABLE I

Cantilever mild steel, length 95 cms dia. 1.27 cms.

Hammers

Hammer A .	Brass, weight, 285.5 gms 'mass ratio' 3.29 radius of curvature at contact surface 2 cms
Hammer B	mild steel, other specifications same as Hammer A
Hammer C	Aluminium, weight 108 gms rad at contact surface 2 cms
Hammer D	mild steel, weight same as Hammer C, radius of curvature at contact surface 1 cm

Hammer velocity before impact cms/sec	A 32.75	A 47.36	A 79.21	A 94.73	A 65	B 65	C 65	D 65
Striking distance (cms)	$1-\left(\frac{v_t}{v_0}\right)^2$							
95	6751	.6636	6519	.6636	6636	6975	9375	.9159
90	.8100	8151	8100	8151	8151	.7975	9121	9395
85	8701	8675	8270	8319	.8479	8400	9444	9494
80	9039	8976	8775	8775	8775	8556	9450	9560
75	9159	9100	8844	8841	9010	8775	9804	.9676
70	9405	9159	9100	9040	9324	.8976	9919	9744
65	.9216	9375	.9160	9160	.9100	.8911	.9933	.9890
60	.9600	.9572	8976	.9100	9100	.8844	.9718	9639
55	.9900	.9879	9801	9831	.9831	.9856	.9600	.9694
50	9780	9600	9600	9600	9600	9744	.9984	.9994
45	.9560	9159	9560	9516	.9448	9560	9920	9980
40	9350	.9324	9216	9324	.9160	9324	.9600	9490
35	8844	8844	.8844	8844	.8775	8775	.9450	9842
30	—	—	—	—	—	.8140	.9906	9958
47.5	(mid-pt)	—	—	—	—	.9663	—	—

TABLE II

Cantilever : same as given in Table I. struck at free end
Hammer : mild steel, velocity before impact 65 cms/sec

Mass ratio	1	1 88	3 29	3 76	8 7
$1-\left(\frac{v_t}{v_0}\right)^2$	3916	.5914	6975	.7286	9159

The curves in Fig. 1 are for the same striking velocity and for hammers A, B, C, and D, respectively. It is found that the value of $1-\left(\frac{V_t}{V_0}\right)^2$ discontinuously fluctuates with striking distances The difference in the loss of energy

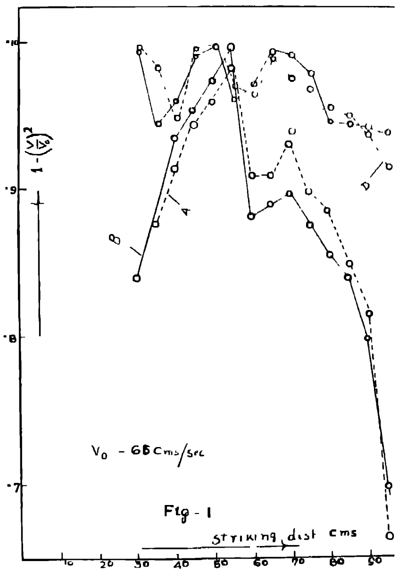

Fig 1

for particular struck point is very small for hammers of same mass but different elastic constant. Further compairing curves A and B, it is found that the nature of the fluctuation of the ratio of the loss of energy to the initial energy of the hammer with the striking distances appears to be similar. But for hammers C and D, this fluctuation is not similar. This shows that the radius of curveture of the hammer at the contact surface plays an important part in this nature of fluctuation. Further from curves A and B, we find that the loss of energy is maximum at the particular struck point (55 cms.) this shows that the energy of bending is maximum at this struck point. But from curves C and D, it is found that this point shifts towards shorter length, (50 cms) as the mass of the hammer decreases.

Fig 2 shows the variation of loss of energy to initial energy of the hammer with 'mass ratio' for the same struck point. It is found that the loss of energy tends to a constant maximum value as the 'mass ratio' increases.

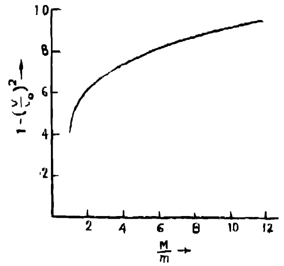

Fig 2.

For the cantilever, struck by hammer B at 47.5 cms (mid. -pt) the calculated value of the quantity $1-\left(\frac{V_t}{v_0}\right)^2$ as per eqn 3, is .9513 and the experimental value (table 1) is .9663 Again for the same hammer, striking at 80 cms., the calculated value of the ratio of loss of energy to initial energy is .8400 and the experimental value is .8556 These are very good agreements.

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